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Analysis of Lubrication for Controlled-Crown Press Rolls: A Parametric Study

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ANALYSIS OF LUBRICATION FOR CONTROLLED-CROWN PRESS ROLLS: A PARAMETRIC STUDY

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ABSTRACT

The objective of this study was to characterize the lubrication flow behavior, and the performance of a crown-compensated (CC) press roll for various operating conditions. The roll rotates counterclockwise, and an external force applied to the top of its internal hydrostatic shoe will cause a pressurized oil to be injected through a series of capillary tubes within the shoe, to transmit load and provide lubrication by producing an oil film between the bottom surface of the shoe and the inside surface of the roll. Because of the forces applied to the shoe and the counterclockwise rotation of the roll, the shoe will have a small vertical displacement and a small clockwise rotation, and under a steady state condition, it will reach an equilibrium position. By generating mathematical expressions for various regions of the roll and the shoe, coupling the Hagen-Poiseuille equation for laminar flow through capillary tubes with the expressions for a viscous flow in a curved-wall channel, and balancing all forces and moments applied to the internal shoe, a system of nonlinear transcendental equations was produced and solved by a numerical scheme. The predicted values of lubrication film thickness, mass flow rate, and

mechanical power required to operate the roll at various applied loads and roll speeds were compared with experimental data provided by the manufacturer, and a good agreement was obtained. Also, a parametric study was performed to determine relative influence of lubricant viscosity, shoe radius, and shoe angle on the predicted quantities, and it was determined that the oil viscosity had a profound effect on the performance of the press roll. The model predicted that operation of the press rolls at relatively higher loads can be achieved without significantly increasing the requirement for mechanical power.

INTRODUCTION

Shown in Fig. 1a is a crown-compensated (CC) extended-nip press roll proposed for impulse drying of a wet sheet which is currently being developed at the Institute of Paper Science and Technology (IPST). Previous studies have demonstrated that implementation of this technique will result in significant energy savings, and improved paper properties (1,2). In this process, wet paper sheets transported on special felt enter an extended nip of the impulse dryer at location A, in Fig. 1a, and leave the nip at point C.

The plasma spray coated roll shell heated by induction heating in region D-E contacts the wet sheet at a surface temperature in the range of 150°C to 250°C. Steam formed at the interface between the roll surface and the sheet which increases the hydraulic pressure gradient, increases the sheet temperature, thus, reduces the water viscosity within the sheet, and results in a greater conforma-

bility for the heated fibers. These effects tends to increase water removal, increase apparent sheet density, and thereby increase the strength of the sheet. Although significant progress has been made towards the development and fundamental understanding of the impulse drying process, the issue of roll surface long-term durability has to be further investigated, before successful commercialization of this technology. Ongoing research at IPST has focused on methods to eliminate catastrophic coating failure and increase its long-term durability. As a first step in prediction of stress distribution within the roll coating, this analytical model was under taken.

As roll rotates counterclockwise (Fig. 1a), an external force is applied to the top of its internal hydrostatic shoe causing a pressurized oil to be injected through a series of capillary tubes within the shoe to transfer load and providing lubrication by producing an oil film between the bottom surface of the shoe and the inside surface of the roll. The oil also acts as a heat sink/source (depending on the magnitude of viscous heat dissipation) for heat transfer between the oil and the inner surface of the roll. Because of the forces applied to the shoe and the counterclockwise rotation of the roll, the shoe will have a small vertical displacement and a small clockwise rotation, and under a steady state condition, it will reach an equilibrium position. The main objective of the analysis was to predict the pressure distribution at the interface of the hydrostatic shoe and the internal roll surface which serve as a boundary condition for a future finite element analysis. Additional objectives of this study were to predict the lubrication

flow characteristics and the performance of a crown-compensated (CC) press roll, and to explore various design, physical, and operating parameters.

ANALYTICAL MODEL

The model analyzes the lubrication flow problem in the channel formed by the bottom surface of the shoe and the inner surface of the roll (Fig. 1b) for a steady state condition, and fixed operating parameters of the applied load and the roll speed. Under this condition the shoe has moved to a position in which a rip located at the top of the shoe has contacted the wall of the confinement shaft of the shoe (Fig. 1). Because of the counterclockwise rotation of the roll, viscous nature of the lubricant, and hydrodynamic forces, the shoe then turns clockwise through an angle ψ , and also executes a vertical motion until reaches an equilibrium position. This model applies the necessary conditions for the static equilibrium (e.g., balancing horizontal and vertical components of forces exerted on the shoe as well as balancing moments of these forces with respect to a point such as B) to determine the magnitude of angular rotation (ψ) and the vertical position of the shoe at point B (d_0), under a specific operating condition. The equilibrium position of the shoe is completely specified by these two independent variables. Since a detail description of the analytical model has been given elsewhere (3,4) in this section, only a summary of the method for model development will be presented. Also, the mathematical expressions for the lubricant velocity profiles, mass flow rate, and forces exerted by the lubricant to the shoe as they correspond to only one re-

gion of the right-hand channel (the region under NC in Fig. 1a) will be listed.

The curved-wall channel formed by the bottom surface of the shoe and the inner surface of the roll, is approximated by a planar walled convergent channel or wedge. In order to solve the steady flow problem in that channel, first the geometry of the channel is specified in terms of known geometric quantities, such as the radii of the shoe and the roll, as well as unknown variables, such as the base thickness of the channel (d_0) and the angular rotation of the shoe (ψ). Then the Hagen-Poiseuille equation for laminar flow through capillary tubes is coupled with the expressions for a viscous flow in a curved-wall channel, and mathematical equations for all the forces exerted by the lubricant on the bottom surface shoe are developed. Finally, the unknown variables of d_0 and ψ are determined by numerical solution of the set of non-linear equilibrium equations which are obtained by balancing all forces and moments applied to the internal shoe for each operating condition of the roll speed and the applied load. Once ψ and d_0 are determined, it is then possible to compute all the geometrical quantities as well as the pressures, mass flow rates, and the velocity fields in the subchannels for each operating condition. Expressions for the tangential and normal forces exerted by the lubricating oil, both on the bottom surface of the shoe as well as on the inside surface of the roll, are also computed, and these results are used to compute the power required to operate the CC roll. In order to validate the model, the predicted values of lubrication film thickness, mass flow rate, and mechanical power required to operate the roll

are compared with experimental data provided by the manufacturer, Beloit Corporation (5). Also, in this study a parametric analysis is performed to determine relative influence of some of the design and parametric variables on the predicted quantities.

The velocity fields are two-dimensional and are obtained by imposing the standard lubrication theory assumption of pseudo-plane Couette flow (6,7). In the region of the right-hand subchannel which is located to the right of the recess ($l \leq x \leq L$), the expression for the velocity field is:

$$u(x, y) = \frac{1}{2\mu} C(x) y [d(x) - y] + s [1 - yd^{-1}(x)] \quad (1)$$

where, μ is the dynamic viscosity of the lubricant, $C(x) = -p'(x)$, s is the tangential velocity of the roll evaluated at its inner surface, $d(x)$ is the thickness of the lubricant at a point along the channel, $l \leq x \leq L$, and $0 \leq y \leq d(x)$. The velocity field $u(x, y)$ satisfies $u(x, 0) = s$, $u(x, d(x)) = 0$. Integration of Eq. 1 results in an expression for the mass flow rate passing through the right-hand channel in terms of an unknown lubricant pressure at the recess (\tilde{p}) and the geometry of the channel. However, this flow equation has to be coupled with another expression for the lubricant flow through a capillary. Assuming Hagen-Poiseuille flow in the capillaries, which are idealized to be circular cylindrical tubes of length \tilde{l}_{eff} and radius \tilde{R}_{eff} , the volumetric flow rate through a capillary which feed the lubricant into the right-hand subchannel is:

$$\dot{q}_c = \pi \tilde{R}_{eff}^4 (p_{sh} - \tilde{p}) / (8\mu \tilde{l}_{eff}) \quad (2)$$

where, p_{sh} and \tilde{p} are the lubricant pressures at the top of the shoe and at the recess, re-

spectively (6). Relating this volumetric flow rate to the mass flow rate obtained from Eq. 1, the following expression which is independent of \tilde{p} can be generated:

$$\frac{\dot{m}}{\rho} = \frac{(p_{sh} - p_{atm}) + \frac{6\mu s}{\tan \beta} \left(\frac{\lambda}{\delta}\right)}{\left[\frac{6\mu}{(\tan \beta)\delta} + \left(\frac{w_{sh}}{n_c}\right) \frac{8\mu \tilde{l}_{eff}}{\pi \tilde{R}_{eff}^4}\right]} \quad (3)$$

where, ρ is the density of the lubricant, p_{atm} is the atmospheric pressure, β is the angle between the upper and the lower walls of the channel, λ and δ are geometric variables related to the thicknesses of the channel at the end of the recess and at the end of the channel, w_{sh} is the cross machine direction width of the shoe, and n_c is the number of capillaries in the cross machine direction. Similar equations can be obtained for the left-hand subchannel, except that the flow direction in this channel is in the opposite direction of the roll rotation. The expressions obtained for the velocity profile and the mass flow rate are used to compute the pressure distribution along the channel, $P(x)$, and then the normal and tangential forces exerted by the lubricant on the bottom surface of the shoe and the inside surface of the roll. The resulting expressions are then used to set up the equilibrium equations which serve to determine ψ and d_0 in terms of the other parameters in the model. The normal force exerted on the shoe (per unit width of the shoe in the cross machine direction) in this region ($l \leq x \leq L$) is computed from:

$$N_{sh} = - \int_l^L [p(x) - \tilde{p}] dx \quad (4)$$

For the tangential forces acting on the bottom surface of the shoe, the following rela-

tion is used:

$$\mathcal{T}_{sh} = -\mu \int_l^L \left[\frac{\partial u}{\partial x}(x, y) \Big|_{y=d(x)} \right] dx \quad (5)$$

Evaluating Eq. 5 at $y = 0$, will result in the tangential forces acting on the inner surface of the roll (\mathcal{T}_{rl}). Figure 2 shows the components of the various forces acting on the bottom surface of the shoe in the right subchannel. Under equilibrium condition, forces applied on the shoe along horizontal and vertical directions, as well as moment of forces about a point, should be in balance. From the three equations of equilibrium, a system of two coupled nonlinear equations are developed which are solve numerically (8) to determine the variables d_0 and ψ for each operation conditions. The mechanical power required to operate the roll is then determined from:

$$W = (\mathcal{T}_{rl})_{tot} s \quad (6)$$

where, W is the mechanical power per unit cross machine direction width of the shoe, $(\mathcal{T}_{rl})_{tot}$ is the algebraic sum of all the shear forces exerted by the lubricant on the inner surface of the roll in all regions of the left-hand and the right-hand channels, and s is the tangential speed of the roll evaluated at the inner surface of the roll.

RESULTS AND DISCUSSION

A. Code Validation

For validation of the analytical model, first the results were compared with the experimental data provided by the Beloit Corporation for a laboratory scale shoe/roll configuration in which the shoe radius (R_s) was

171.32 mm (6.745 in) and the roll inner radius (R) was 171.50 mm (6.76 in). This shoe was subjected to outer surface roll speeds of 305-914 m/min (1000-3000 ft/min) and applied loads of 35-175 KN/m (200-1000 PLI). The lubricant viscosity and density values were assumed to be 56 centipoise and 873 Kg/m³, respectively. These properties correspond to the lubricant employed in experiments conducted by the Beloit Corporation. A constant lubricant temperature of 57°C was assumed. Since the speed of the roll and the load applied to the top of the shoe are two of the most important input parameters which can be controlled by an operator for specific design conditions, all the calculated values were obtained at a function of these two parameters.

The lubricant thicknesses at points O and N (Fig. 2) are referred to as the leading edge thickness (L.E.) and the trailing edge thickness (T.E.), respectively. As the roll rotates in a counter-clockwise direction, the L.E. (point O) experiences the roll shell before the T.E. (point N). Shown in Table 1 is a comparison between the predicted and experimental values for the L.E. and T.E. thicknesses, and their average values at two outer surface roll speeds and three applied loads. Under the influence of the applied load and the hydrodynamic forces, the shoe can rotate about its pivot point (point G in Fig. 2) and also can move up or down along the confinement wall. The model and the experiment show that the leading edge thickness is greater than the trailing edge thickness for all the operating conditions; thus, the shoe has turned in a clockwise direction. Also, both methods indicate that the lubricant thicknesses at the L.E. and T.E.

locations increase with an increase in roll speed and decrease with an increase in applied load. At a constant load, the lubricant film thickness increases with an increase in roll speed because of the hydrodynamic effect of the lubricant, which increases the magnitude of the normal forces exerted by the fluid on the bottom surface of the shoe. The behavior of the shoe in this regard is similar to that of an air foil, whose lift increases with air foil speed. Table 1 indicates that there was a good agreement between the model and the experiment for the average values of the L.E. and the T.E. thicknesses.

Shown in Fig. 3 is a comparison of the total volumetric flow rates in the channel as predicted by the model and as measured for a roll outer surface speed of 610 m/min. This figure indicates that at a fixed roll speed, an increase in applied load results in a linear increase in the lubricant flow rate. For this operating condition, the volumetric flow rates predicted by the model were only slightly smaller than the experimental data. Figure 4 compares the mechanical power required to operate the roll as predicted by the model and as measured by the Beloit Corporation for the two roll outer surface speeds of 305 m/min and 610 m/min. Both the model and experiment predicted that for this laboratory scale shoe/roll configuration, the mechanical power required to operate the roll increases with applied load; however, the increase was small at the lower roll speed. In general, the predicted results for the lubricant thickness, flow rate, and mechanical power were in good qualitative agreement with the corresponding experimental data, and the relatively small difference between their numerical values may be attributed to

some simplifying assumptions used in the analytical model, as well as to the tolerance and error associated with the experimental data. Some of the assumptions used in the model are as follows:

1. A small portion of the bottom surface of the shoe at each end was tapered; this section was not considered in the model.

2. The arcs describing the bottom surfaces of the shoe and the roll were approximated by secant lines. This is a good approximation for a roll/shoe configuration with a relatively large size diameter. However, as the diameter of the shoe/roll decreases, the error associated with this secant line approximation increases.

3. All machine dimensions, including the dimensions of both the shoe and roll, are accurate only to within 0.002 inch.

4. In the experiment, the oil which exits the channel on the right is not skimmed off, but rather, re-enters the channel on the left.

5. The results presented are based on the assumption of a constant oil viscosity (the viscosity at the lubricant inlet temperature). Because of viscous dissipation, the average temperature in the oil will be higher at higher speeds and loads than it is at lower speeds and loads, and at any fixed speed and load the temperature in the oil varies (spatially) throughout the channel. Also the lubricant property data show that the oil viscosity significantly decreases with increasing oil temperature. As will be shown later, the viscosity can have a major influence in some of the predicted quantities. Therefore,

a temperature dependent viscosity should be considered in future studies.

In spite of these simplifying assumptions, comparison of the model with the experiment indicates that this model can provide useful information about the behavior of the press roll for various design and operating conditions. Since the credibility of this analytical model was considered to be proven by the comparison with the experimental data, additional analyses were performed for a commercial size roll

B. Analysis of a Commercial Size Press Roll

The analytical model also was employed for a commercial size press roll in which the roll and its internal shoe were machined to the same radius of 508.13 mm ($R_s = R = 20.005$). For this analysis, the shoe was subjected to loads in the range of 175-1751 KN/m (1000-10,000 PLI) and outer surface roll speeds of 305-1067 m/min (100-3500 ft/min). The wall thickness for this roll was 3.0 inches, which indicates that the tangential speed experienced by the lubricant at the inner surface of the roll is approximately 0.85 times less than that of the outer roll surface. Shown in Figs. 5-10 are the predicted results for the commercial size press roll. For all the operating conditions, the pressure distribution along the inside surface of the roll (Fig. 5) is constant at the recess regions and falls off monotonically toward the end of the subchannels. The negative values on the horizontal axis correspond to the left-hand subchannel located on the left of the origin shown in Fig. 2. The pressure exerted by the lubricant on the inner surface

of the roll was slightly greater at the right-hand channel compared with that of the left-hand channel. Not only does pressure profile have an impact on the water removal during wet pressing, but also the thermomechanical stresses developed within the surface coating of an impulse drying press roll may be significantly affected by this pressure distribution. The analytical model demonstrated that, in general, the speed of the roll had only a small impact on the pressure distribution; however, the mechanical power required to operate the roll was significantly influenced by the roll speed (Fig. 6). This figure indicates that for the commercial size roll, the mechanical power is relatively insensitive to the applied load. Thus, for more effective water removal, this roll can be operated at a higher load without any major increase in required power.

The results of the parametric study for this press roll at a fixed tangential speed of 610 m/min and a fixed applied load of 1751 KN/m where the shoe angle, the radius of the shoe, and the lubricant viscosity were used as the geometric/physical parameters are shown in Figs. 7-10. The shoe angle corresponds to the angle between two radial lines emanating from the center of the shoe, and passing from the points B and C (Fig. 2). These parametric analyses were performed to gain more insight about the design and operation of crown-compensated (CC) impulse drying press rolls as well as CC extended nip rolls, and CC calendar rolls. In these analyses, sensitivity of the results with respect to variation of the lubricant viscosity in the range of 1-100 cp, shoe angles of $18.96^\circ - 25^\circ$, and shoe radii of 507.12-508.12 mm were evaluated. The case in which the

lubricant viscosity, the shoe angle, and the shoe radius were 50 cp, 18.96° , and 508.13 mm, respectively, was considered to be the nominal (standard) case, and it was used to determine the impact of the three parameters on the mechanical power, the lubrication flow rate, the angle of rotation of the shoe, and the lubrication thickness. Figure 7 shows the effect of the oil viscosity on the mechanical power and the lubricant mass flow rate, and Fig. 8 shows the effect of the viscosity on the lubrication thicknesses and the recess pressures. The mechanical power increased linearly with an increase in the oil viscosity (Fig. 7), however, an increase in the viscosity from 1 cp to 10 cp resulted in a significant reduction in the lubricant mass flow rate, while an increase from 50 cp to 100 cp did not have a major effect on the mass flow rate. The viscosity had linear effect on the lubricant thickness and the recess pressure (Fig. 8). The effect of the shoe angle on the mechanical power, and the lubricant mass flow rate are shown in Fig. 9, and their effect on the lubricant thickness and rotation angle are shown in Fig. 10. For the magnitudes of the shoe radius analyzed in this study, the lubrication flow characteristics and the behavior of the shoe were not affected by this design variable. In general, for the range of shoe radius analyzed in this study, this parameter did not have any significant impact on the results.

To make a quantitative comparison between the impact of these variables on the operation of the roll, a design/operation tolerance was assumed for each of these parameters (Table 2), and variation of mechanical power, mass flow rate, rotation angle, and lubrication thickness corresponding to these

assumed tolerances were computed. As Table 2 indicates, the oil viscosity has the most significant influence on the lubricant mass flow rate. Comparison of these variations, with the values obtained for the nominal case (table 3) indicates that when the oil viscosity changes because of its temperature change which is resulted from the viscous heat dissipation, all the major quantities can be affected by this parameter. It should be emphasized that in this parametric study, the compound effect of varying two or more parameters simultaneously was not examined. To come up with an optimum operation and design condition, interaction of these variables should also be analyzed. Also, additional studies should be conducted to determine the impact of these variables on the performance of the hydrostatic shoe, using a temperature dependent viscosity, and also, to analyze the influence of the viscous heat dissipation on the thermal performance of the impulse drying press roll.

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TABLE 1 - Predicted and measured lubrication thicknesses (mm) at the leading edge (L.E.) and the trailing edge (T.E.), and their average values as a function of roll speed and applied load.

| Speed (m/min) ⇒ | | 305 | | | 610 | | |
|------------------------|-------------|------------|------------|------------|------------|------------|------------|
| Load (KN/m) ⇒ | | 105 | 140 | 175 | 105 | 140 | 175 |
| Experiment | L.E. | 0.129 | 0.117 | 0.109 | 0.173 | 0.157 | 0.145 |
| | T.E. | 0.104 | 0.096 | 0.093 | 0.112 | 0.102 | 0.099 |
| | Avg. | 0.116 | 0.106 | 0.101 | 0.142 | 0.129 | 0.122 |
| Model | L.E. | 0.257 | 0.247 | 0.240 | 0.336 | 0.300 | 0.268 |
| | T.E. | 0.106 | 0.094 | 0.087 | 0.140 | 0.129 | 0.124 |
| | Avg. | 0.182 | 0.170 | 0.163 | 0.238 | 0.214 | 0.196 |

TABLE 2- Nominal (standard) condition used in the parametric study.

| Input Data for Nominal Condition | Power (hp/in) | Flow (kg/m min) | Rotation (deg.) | Thickness (mm) |
|---|--------------------------|----------------------------|----------------------------|---------------------------|
| Viscosity = 50 cp Shoe Angle = 18.96° Shoe Radius = 508.13 mm | 0.149 | 1312 | 0.059 | 0.394 |

TABLE 3- Comparison of the effect of each parameter.

| VARIABLE (RANGE) | ASSUMED TOLERANCE | Δ Power (hp/in) | Δ Flow (kg/m min) | Δ Rotation (deg.) | Δ Thickness (mm) |
|---|------------------------------|----------------------------|------------------------------|------------------------------|-----------------------------|
| VISCOSITY (1-100 cp) | 5 | 0.015 | -328.1 | 0.0012 | 0.015 |
| SHOE ANGLE (18.96°-25.0°) | 1 | 0.001 | 136.5 | 0.0011 | 0.029 |
| SHOE RADIUS (507.12-508.12 mm) | 0.5 | 0 | 0.12 | 0 | 4E-5 |

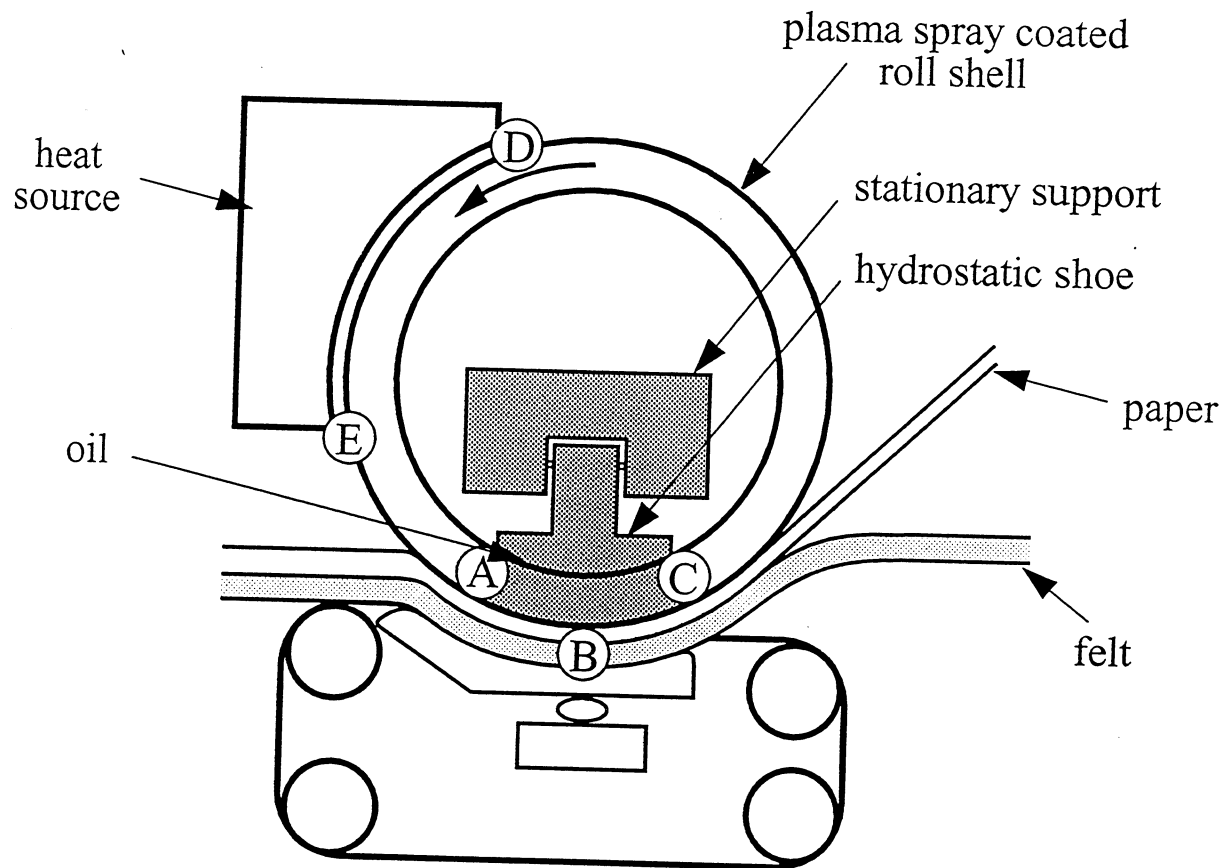


Figure 1a. The crown compensated impulse drying press roll (not shown to scale).

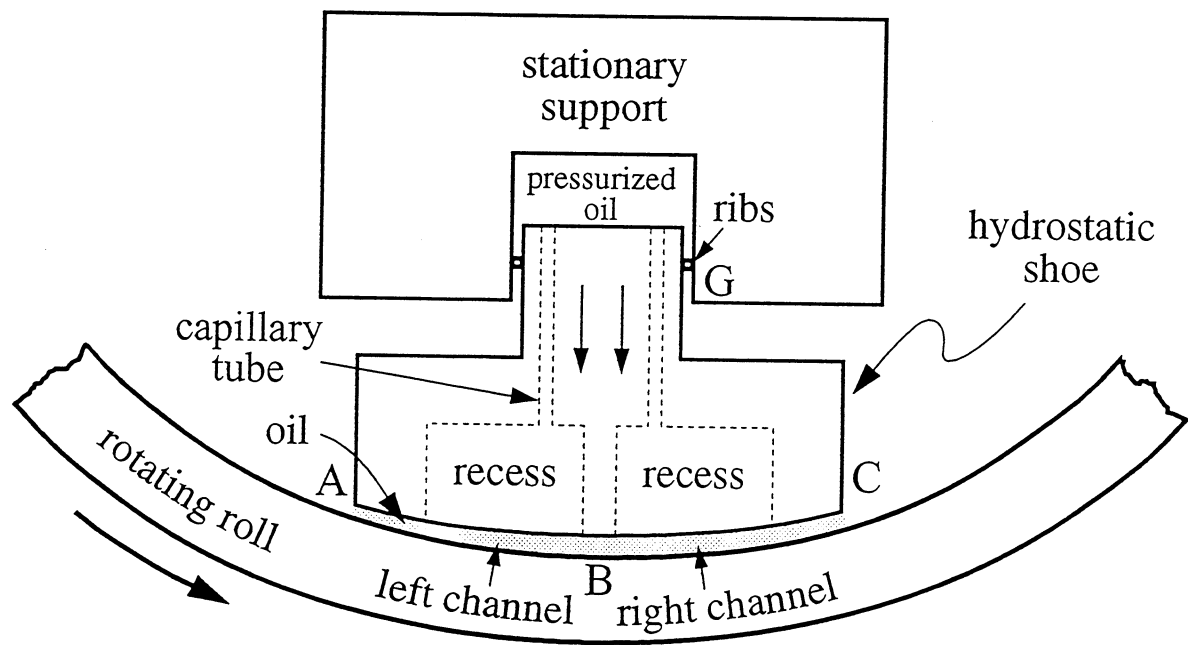


Figure 1b. The hydrostatic shoe and lubrication channels.

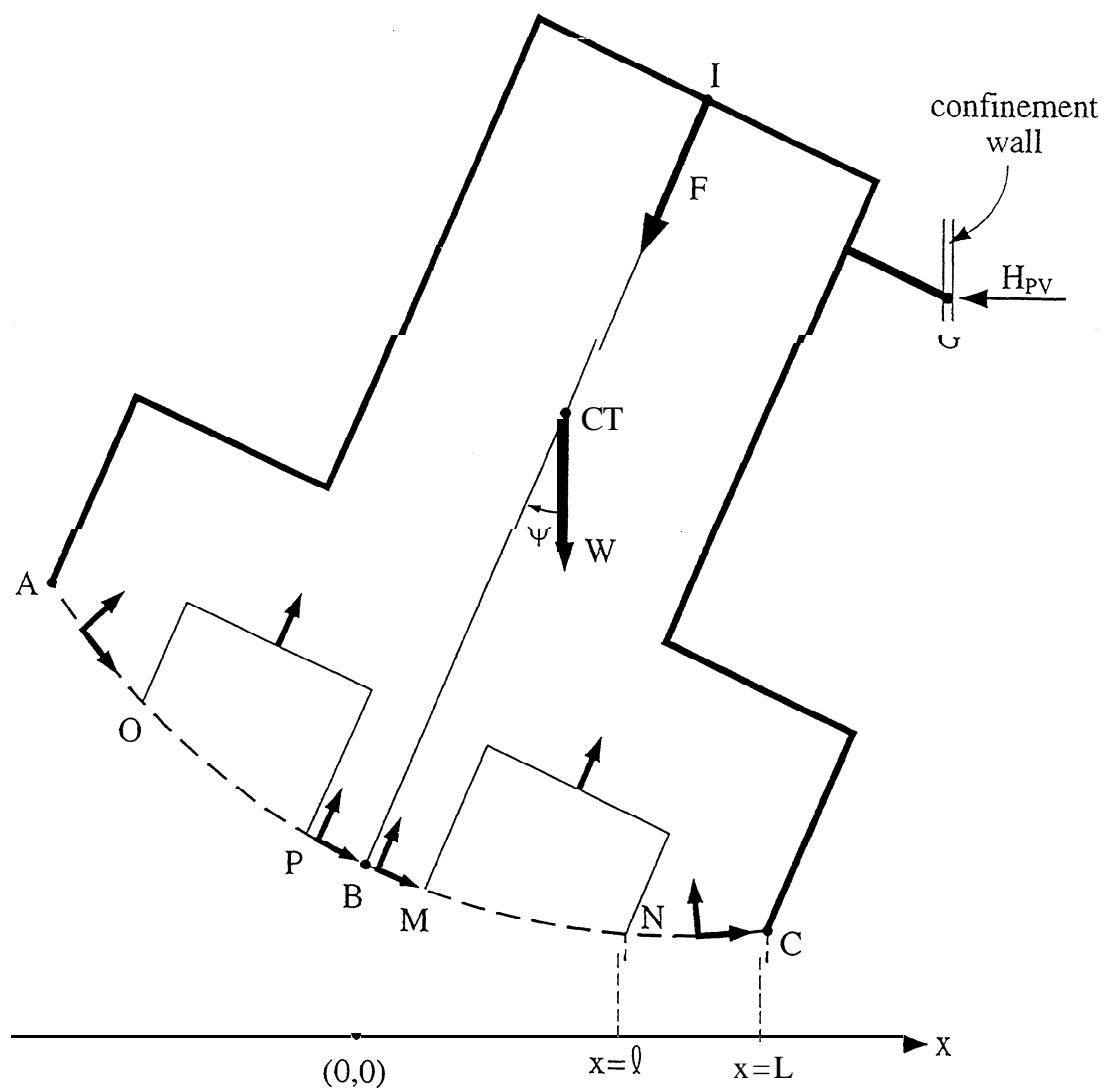


Figure 2. Motion of the hydrostatic shoe and the components of forces acting on it.

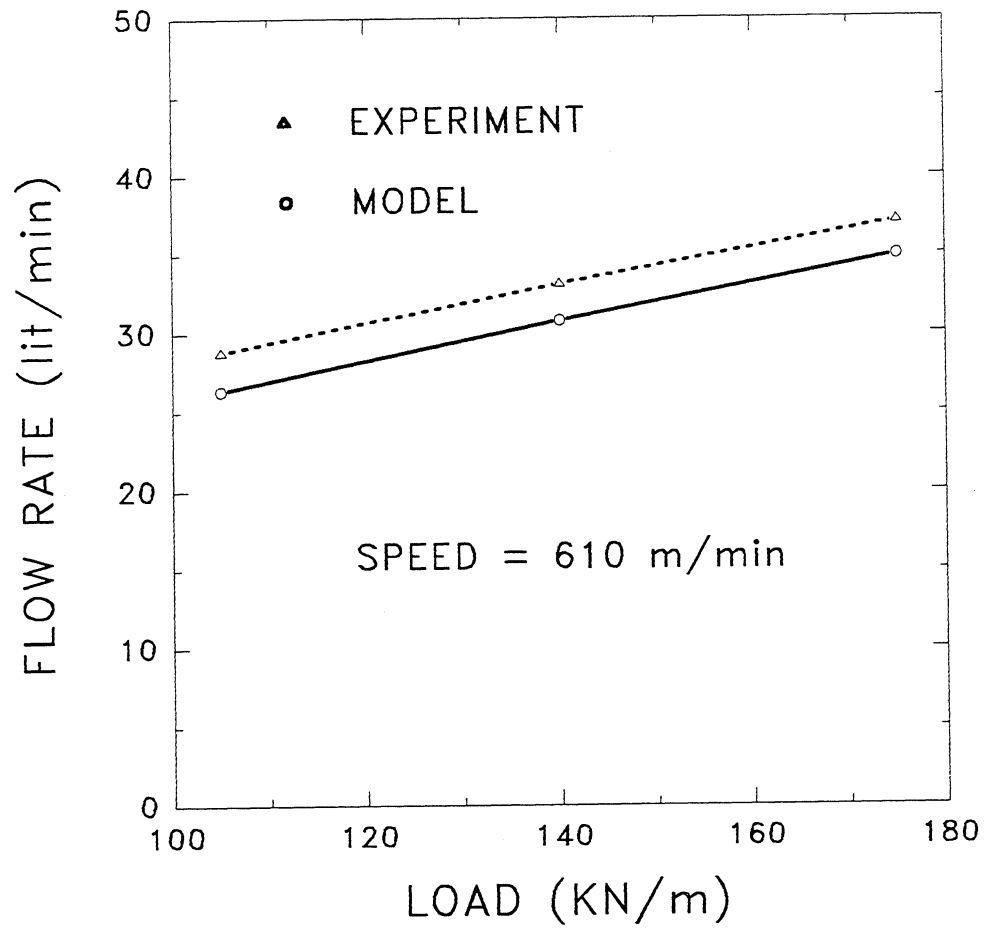


Figure 3. Comparison between the experimental and the calculated lubricant volumetric flow rates.

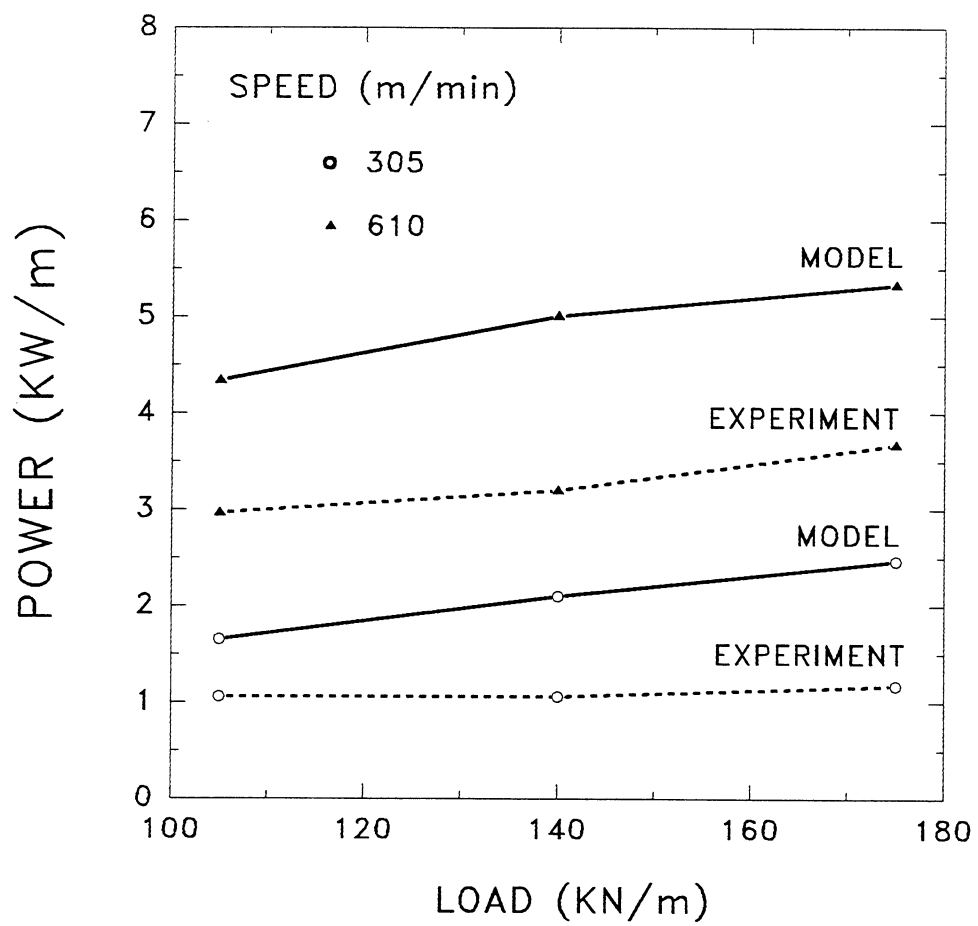


Figure 4. Comparison between the experimental and the calculated mechanical powers.

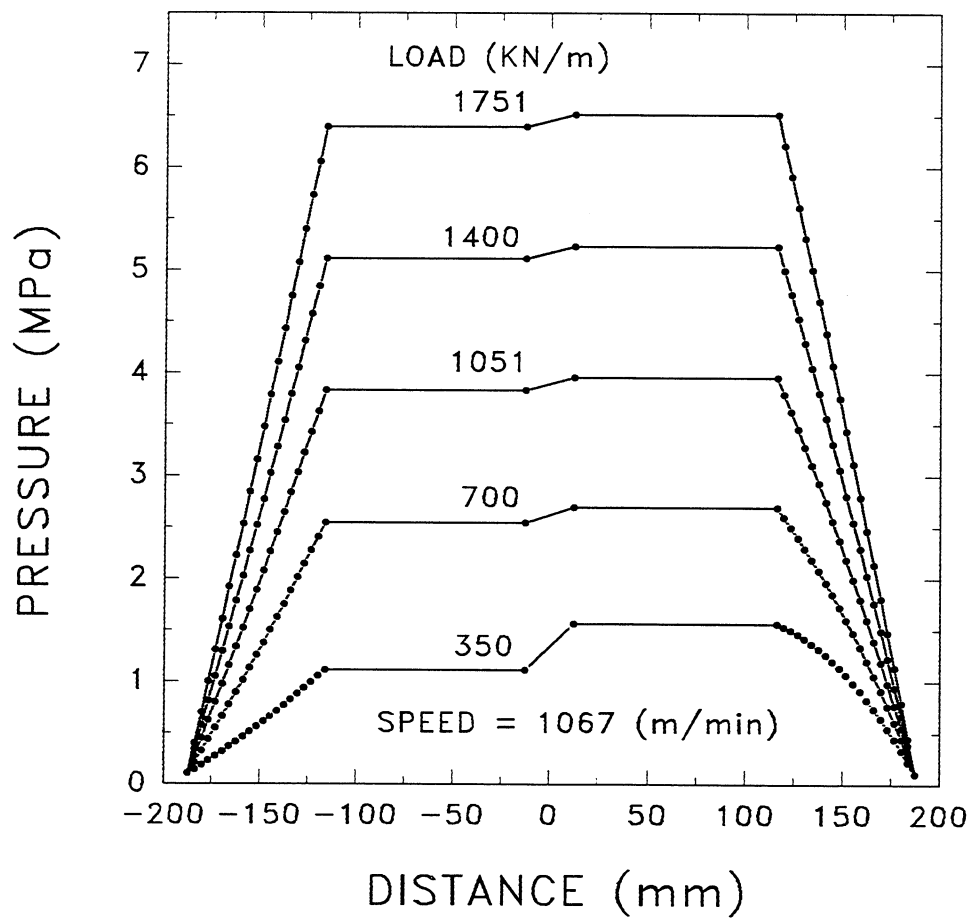


Figure 5. Predicted lubricant pressure distributions for various applied load (commercial size roll).

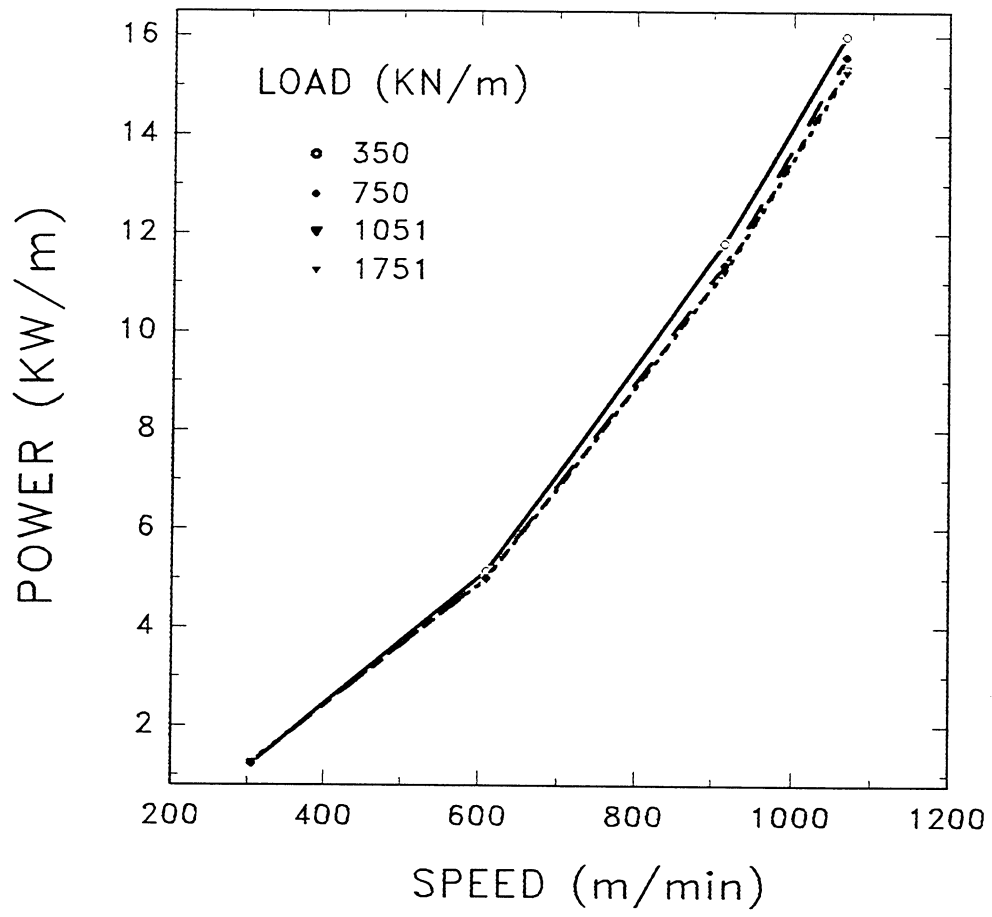


Figure 6. Predicted mechanical power vs. speed for various applied load (commercial size roll).

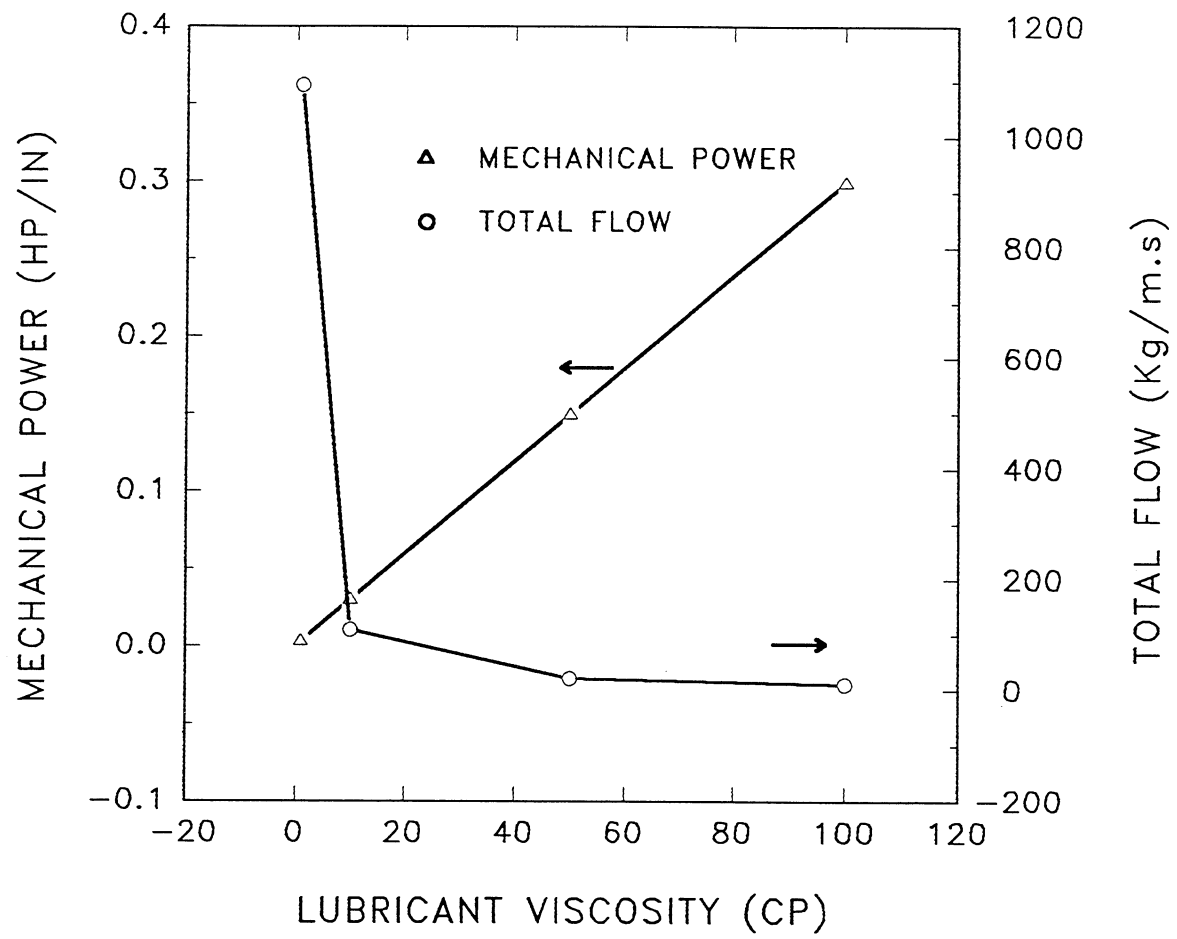


Figure 7. Influence of lubricant viscosity on mechanical power and mass flow rate.

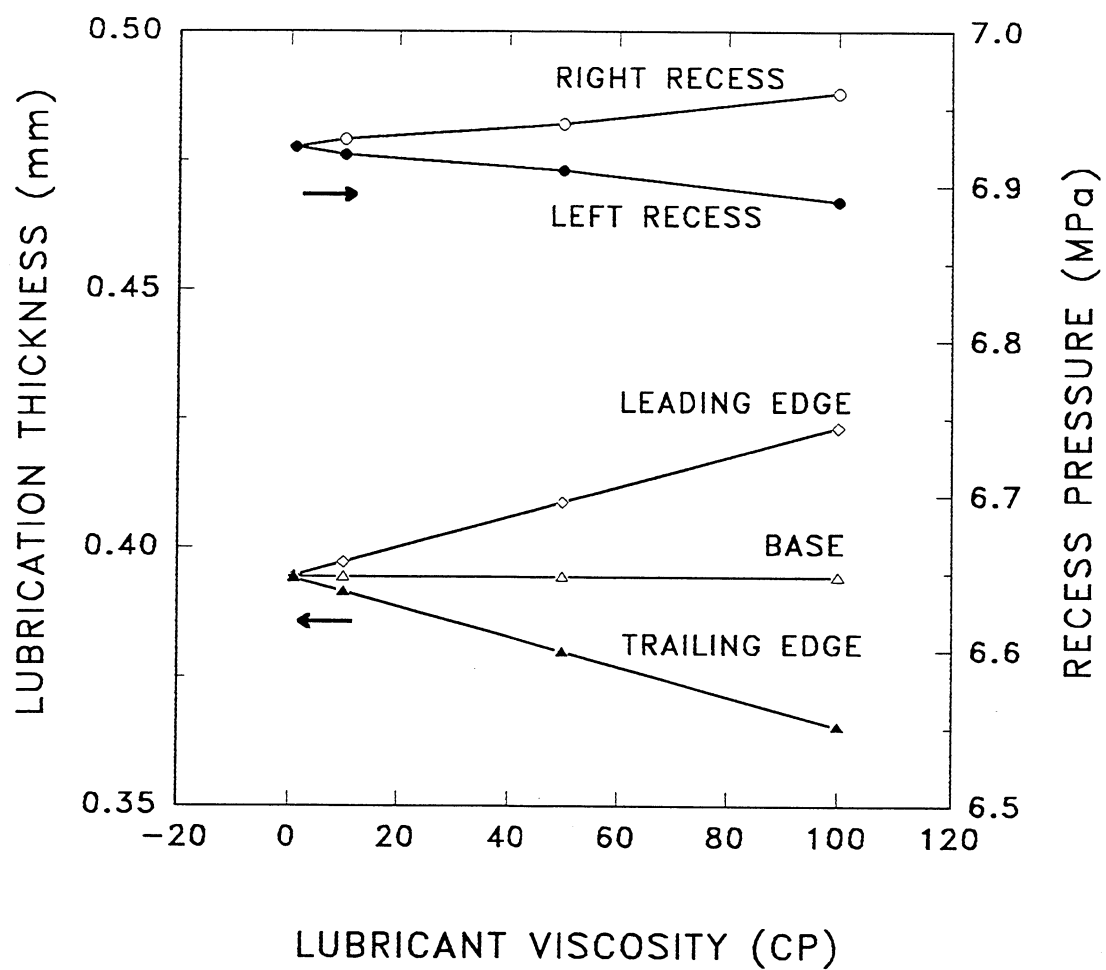


Figure 8. Influence of lubricant viscosity on the lubrication thickness and recess pressure.

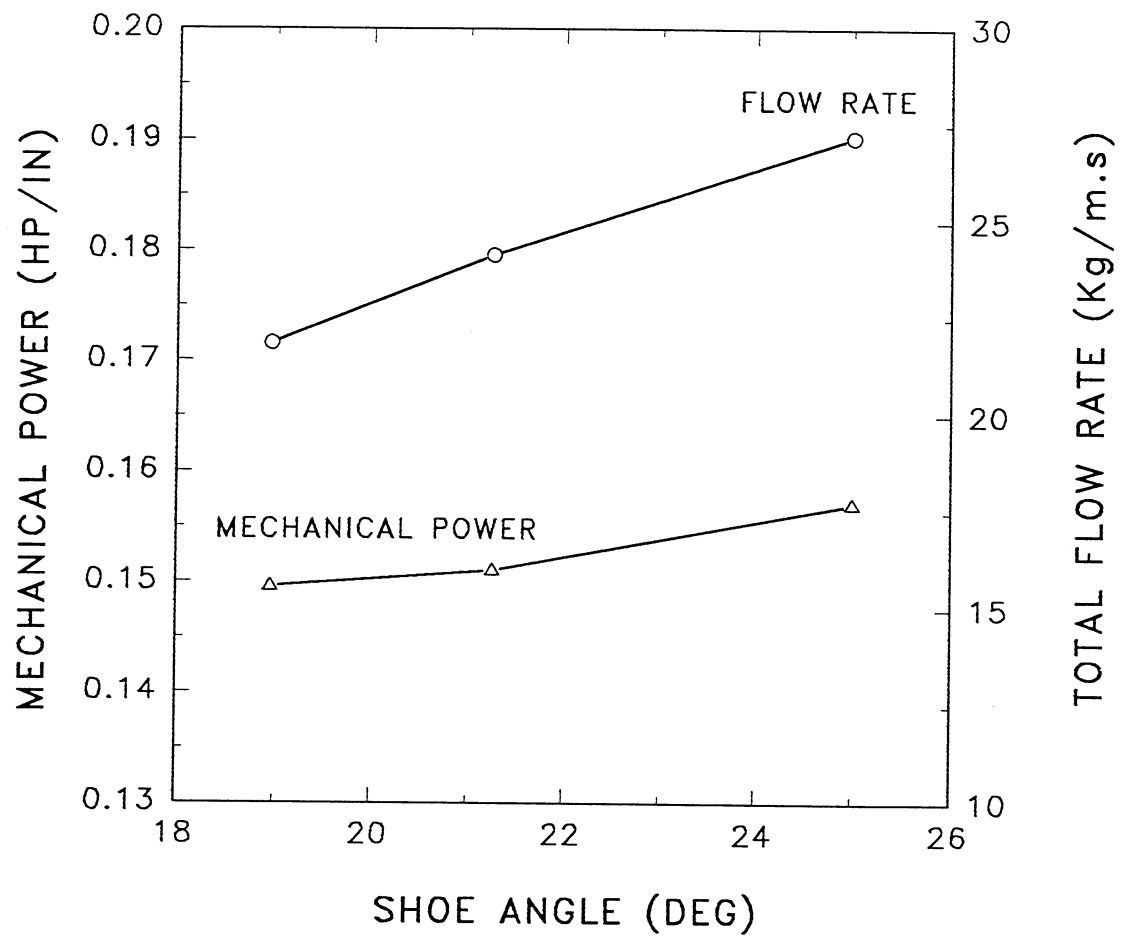


Figure 9. Influence of the shoe angle on mechanical power and lubricant mass flow rate.

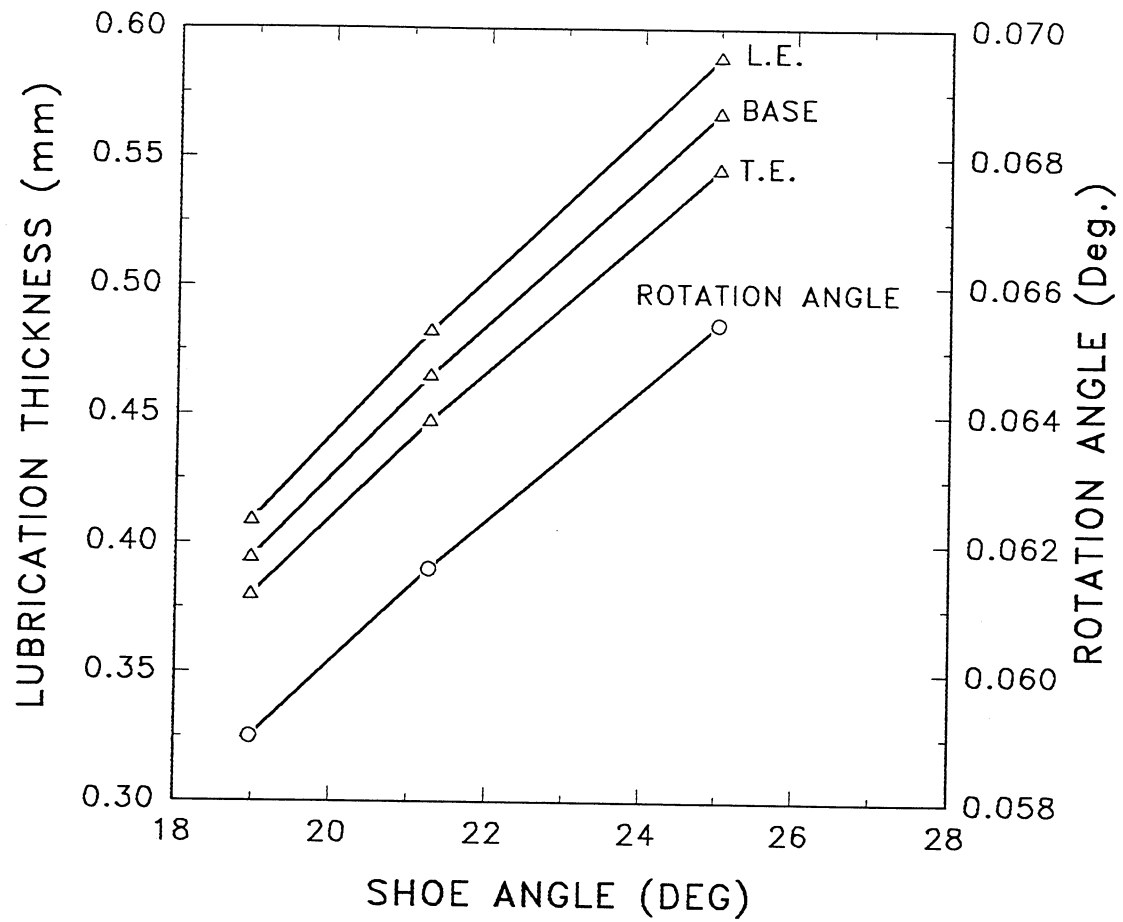


Figure 10. Influence of the shoe angle on lubricant thickness and rotation angle of the shoe.

